Name Solutions

September 19, 2012

ECE 311 Exam 1 Fall 2012

Closed Text and Notes

1) Be sure you have 9 pages.

2) Write only on the question sheets. Show all your work. If you need more room for a particular problem, use the reverse side of the same page.

3) no calculators allowed

4) Write neatly, if your writing is illegible then print.

5) This exam is worth 100 points.

(3 pts) 1. Find the magnitude of $\mathbf{B} = 3\hat{\mathbf{a}}_\rho + 1\,\hat{\mathbf{a}}_\phi - 2\hat{\mathbf{a}}_z$

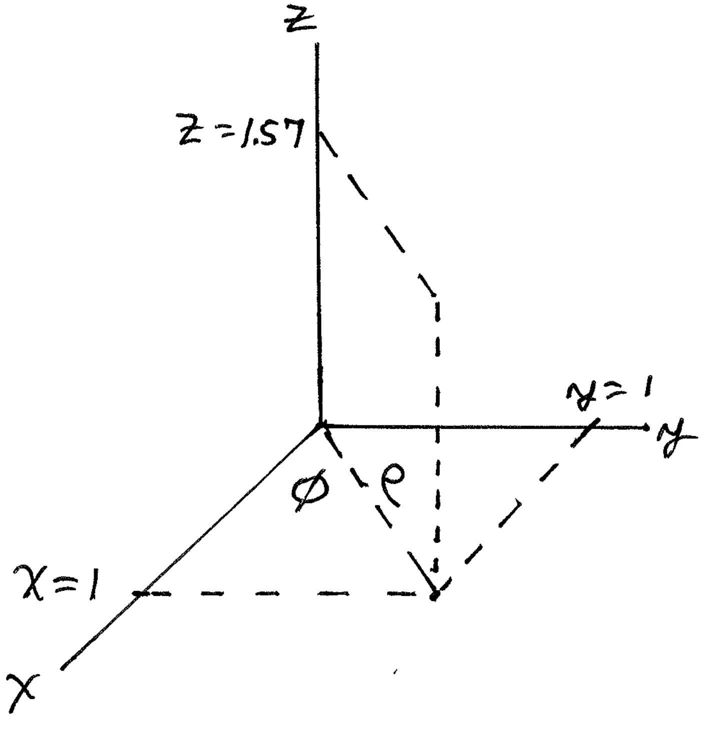
$$B^2 = \vec{B}\cdot\vec{B} = (3\hat{a}_\rho + 1\hat{a}_\phi - 2\hat{a}_z)\cdot(3\hat{a}_\rho + 1\hat{a}_\phi - 2\hat{a}_z)$$

$$= (3)^2 + (1)^2 + (-2)^2$$

$$= 9 + 1 + 4 = 14$$

$$B = \sqrt{14}$$

(3 pts) 2. Convert the point (1, 1, 1.57) in Cartesian to cylindrical coordinates.



$$\rho = \sqrt{x^2+y^2} = \sqrt{1+1} = \sqrt{2}$$

$$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{1}{1} = \frac{\pi}{4}$$

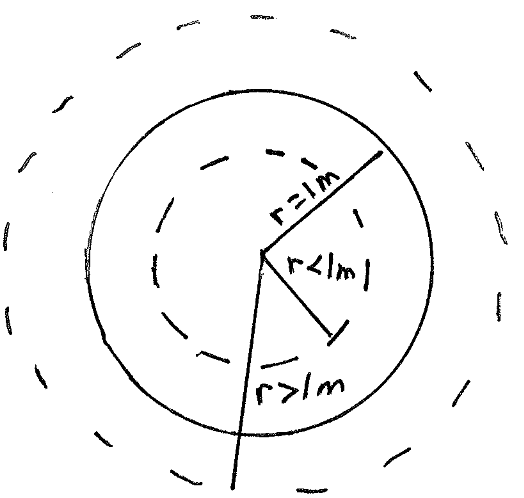
$$z = 1.57$$

$$\left(\sqrt{2}, \frac{\pi}{4}, 1.57\right)$$

(4 pts) 3. What geometry is described by the intersection of the surfaces $\rho = 1\text{m}$ and $\phi = \frac{\pi}{3}$?

infinite straight line

(16 pts) 4. The spherical region r < 1 m has a uniform volume charge density of $\rho_V = 1\frac{C}{m^3}$. Find the electric flux density field everywhere.



$$\rho_v = 1 \frac{C}{m^3} \qquad r < 1m$$
$$= 0 \qquad r > 1m$$

$$\vec{D}(r,\theta,\phi) = D_r(r)\,\hat{a}_r$$

Apply Gauss' Law for the spherical surface r < 1m

$$\oint \vec{D}\cdot\vec{ds} = \int \rho_v\, dv = \rho_v \int dv = \left(1\frac{C}{m^3}\right)\left(\frac{4}{3}\pi r^3\right)$$

$$D_r\, 4\pi r^2 = \frac{4}{3}\pi r^3$$

$$D_r = \frac{\frac{4}{3}\pi r^3}{4\pi r^2} = \frac{r}{3} \qquad \text{for } r < 1m$$

Apply Gauss' law for the spherical surface r > 1m

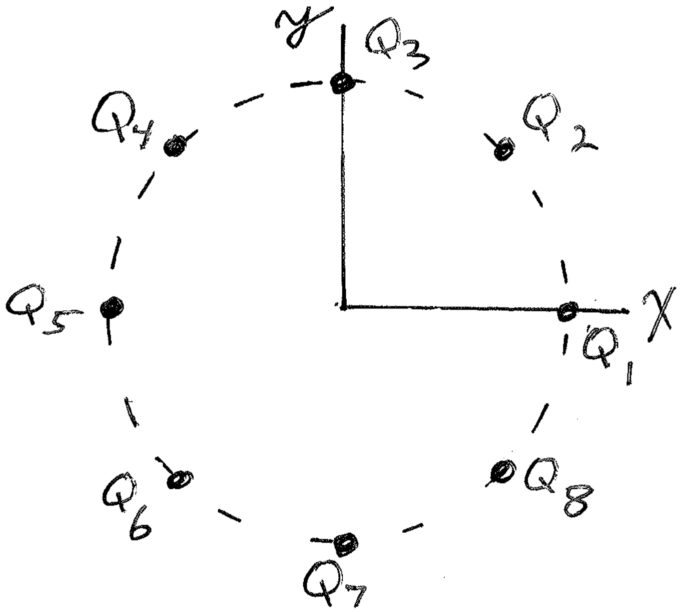
$$\oint \vec{D}\cdot\vec{ds} = \int \rho_v\, dv = \rho_v \text{ (volume of sphere } r = 1m)$$

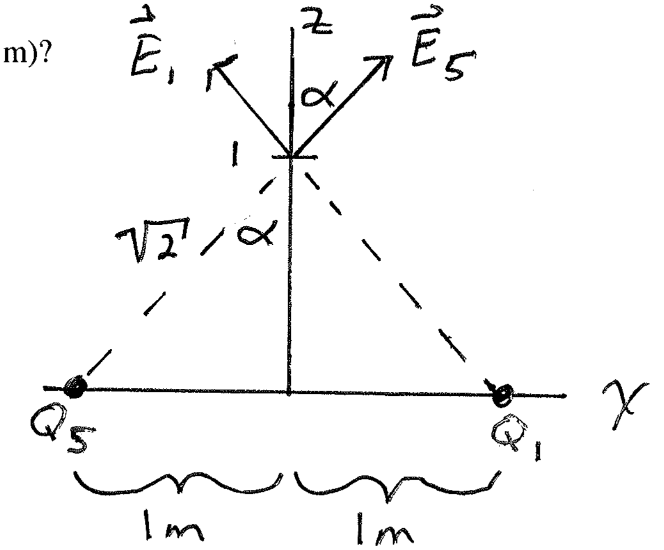
$$D_r\, 4\pi r^2 = \rho_v \frac{4}{3}\pi r^3 = \left(1\frac{C}{m^3}\right)\left(\frac{4}{3}\pi (1m)^3\right) = \frac{4\pi}{3}C$$

$$D_r = \frac{\frac{4}{3}\pi}{4\pi r^2} = \frac{1}{3r^2}$$

$$\vec{D}(r,\theta,\phi) = \frac{r}{3}\hat{a}_r \qquad r < 1m$$
$$= \frac{1}{3r^2}\hat{a}_r \qquad r > 1m$$

(16 pts) 5. Eight $\frac{2}{9}\sqrt{2}\,nC$ charges are equally spaced around a circle of radius 1 m in the z = 0 plane. What is $\mathbf{E}(0,0,1m)$, the electric field intensity at (0, 0, 1m)?





For every pair of charges on opposite sides of the circle the z-components of the $\vec{E}$ field will add and the other components will cancel.

$$E_5 = \frac{Q_5}{4\pi\epsilon_0 R^2}$$

$$= \frac{\frac{2}{9}\sqrt{2}\times 10^{-9}\,C}{4\pi\left(\frac{10^{-9}}{36\pi}\,\frac{F}{m}\right)(\sqrt{2}\,m)^2} = \sqrt{2}\,\frac{C}{Fm} = \sqrt{2}\,\frac{C}{\frac{C}{V}m} = \sqrt{2}\,\frac{V}{m}$$

$$E_{5z} = E_5\cos\alpha = \left(\sqrt{2}\,\frac{V}{m}\right)\left(\frac{1}{\sqrt{2}}\right) = 1\,\frac{V}{m}$$

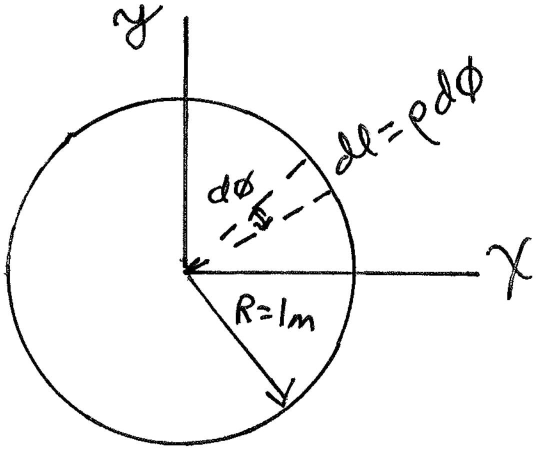
Since the z-components will be the same for all eight charges

$$E_z = 8E_{5z} = 8\left(1\,\frac{V}{m}\right) = 8\,\frac{V}{m}$$

so,

$$\vec{E}(0,0,1m) = 8\,\frac{V}{m}\,\hat{a}_z$$

(16 pts) 6. A ring of radius 1 m and with a line charge density of $\rho_L = \frac{1}{2\pi}\frac{nC}{m}$ is in the z = 0 plane. What is the value of $V(0,0,0)-V(\infty)$?



$$dq = \rho_L dl = \rho_L \rho d\phi$$

$$V(0,0,0) - V(\infty) = \int_0^{2\pi} \frac{\rho_L dl}{4\pi\epsilon_0 R} = \int_0^{2\pi} \frac{\rho_L \rho d\phi}{4\pi\epsilon_0 R}$$

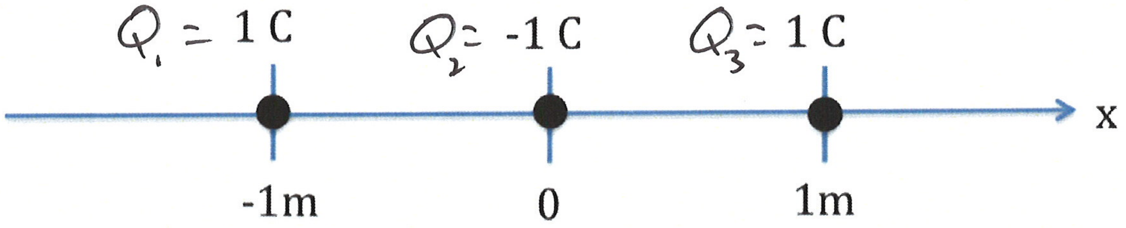
$$= \int_0^{2\pi} \frac{\left(\frac{1}{2\pi} \times 10^{-9} \frac{C}{m}\right)(1m)\, d\phi}{4\pi\left(\frac{10^{-9}}{36\pi}\frac{F}{m}\right)(1m)}$$

$$= \int_0^{2\pi} \frac{9}{2\pi} \frac{C}{F} d\phi = \frac{9}{2\pi}\int_0^{2\pi} d\phi \ V$$

$$= \frac{9}{2\pi} 2\pi \ V$$

$$V(0,0,0) - V(\infty) = 9V$$

(16 pts) 7. Three point charges are aligned on the x-axis as shown.



How much work is required to separate these charges, that is move them infinitely apart from each other?

The amount of work to separate the charges is minus the work to assemble them.

Bring the charges in one at a time. Determine the work done in bringing in each charge and add these works. $W = W_1 + W_2 + W_3$

No work is done in positioning Q_1, $W_1 = 0$

There is now an electric field caused by Q_1 so work will be done positioning Q_2

$$W_2 = Q_2 V_{21} = (1C)\frac{(-1C)}{4\pi\epsilon_0(1m)} = \frac{-1C^2}{4\pi\left(\frac{10^{-9}}{36\pi}\frac{F}{m}\right)1m}$$

$$= -9\times10^{+9}\,J$$

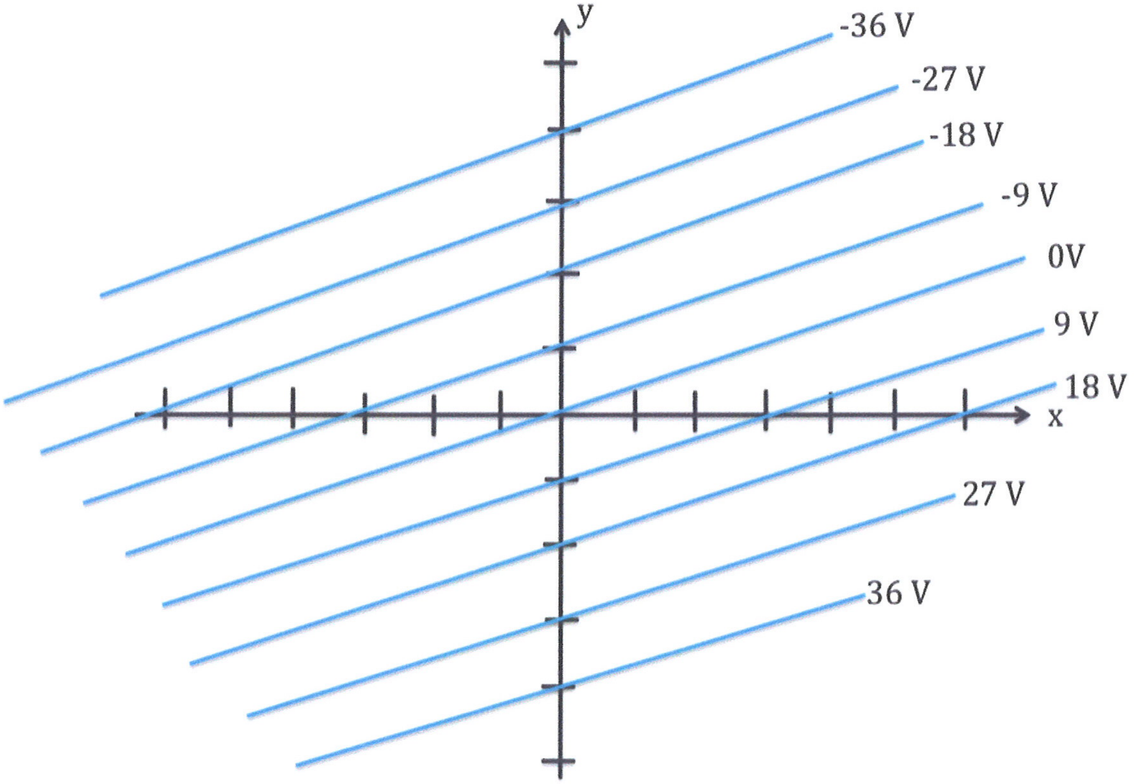
$$W_3 = Q_3 V_{31} + Q_3 V_{32} = \frac{(1C)^2}{4\pi\epsilon_0(2m)} - \frac{(1C)^2}{4\pi\epsilon_0(1m)}$$

$$= 4.5\times10^{+9}J - 9\times10^{+9}J = -4.5\times10^{+9}\,J$$

$$W = W_1 + W_2 + W_3 = 0 - 9\times10^{+9}J - 4.5\times10^{+9}J = -13.5\times10^{+9}J$$

So to separate the charges requires 1.35×10^{10} J of work

(12 pts) 8. All of space is filled with infinite planes of equipotential that are parallel to the z-axis and spaced as shown. The ticks on the x- and y-axis represent 1 m steps. The potential step between the shown equipotential surfaces is 9 V. Determine the electric field intensity.



$$E_x \hat{a}_x = -\frac{\partial V}{\partial x}\hat{a}_x = -\frac{9V}{3m}\hat{a}_x = -3\frac{V}{m}\hat{a}_x$$

$$E_y \hat{a}_y = -\frac{\partial V}{\partial y}\hat{a}_y = -\frac{-9V}{1m}\hat{a}_y = 9\frac{V}{m}\hat{a}_y$$

$$E_z \hat{a}_z = -\frac{\partial V}{\partial z}\hat{a}_z = 0$$

$$\vec{E} = \left(-3\hat{a}_x + 9\hat{a}_y\right)\frac{V}{m}$$

(14 pts) 9. Indicate whether the following statements are true or false.

The equipotential surfaces for an infinite line of charge are cylinders of infinite length with the line charge down the center of these cylinders.	[✓ True]	False
In cylindrical coordinates $\hat{\mathbf{a}}_\rho$ and $\hat{\mathbf{a}}_\phi$ are normal to the surface $\rho=2m$.	True	[✓ False]
By saying the static electric field intensity is conservative means the work done on moving a charge around a closed path is zero.	[✓ True]	False
A solid metal sphere of radius 1 m has +1 C of charge on it. The potential at the center of the sphere is zero because there can be no electric field inside the sphere.	True	[✓ False]
In a static situation, the electric field intensity is always normal to the surface of a conductor.	[✓ True]	False
The potential at infinity is always zero.	True	[✓ False]
If the electric field intensity is in the $\hat{\mathbf{a}}_x$ direction and you are walking in the $-\hat{\mathbf{a}}_x$ direction, you are walking towards regions of higher potential.	[✓ True]	False